Understanding ocular wavefront aberration – Part 2

Professor Neil Charman concludes his discussion of wavefront aberration analysis with a few cautionary words. Equations and Tables from Part 1 (OPTICIAN, July 8, page 24) are referred to

THE FULL SPHERO-CYLINDRICAL CORRECTION

Having considered the spherical defocus, we can now turn to the problem of deriving the full sphero-cylindrical correction from the total wavefront errors. If we examine the Zernike polynomials in Table 1 we see that

$Z_2^{-2} = \sqrt{6}\rho^2\sin2\theta$

Since $\sin2\theta = +1$ when $\theta = 45$ or 225, $= -1$ when $\theta = 135, 315$, with zeros at $\theta = 0, 90, 180, 270$ degrees, and there is a ρ^2 dependence, we can see from the same basic sag formula ideas as were used in the spherical defocus case that this polynomial represents a crossed-cylinder wavefront with axes at 45/135. Taking account of the fact that many of the higher-order polynomials (Z_4^{-2}, Z_6^{-2} etc) also contain terms in $\rho^2\sin2\theta$ and proceeding by analogy with the spherical case by considering the curvatures in the principal meridians, the full, paraxial, crossed-cylinder correction in the 45 and 135 meridians can be found as:

$J_{45} = -(2\sqrt{6}.C_2^{-2} - 6\sqrt{10}.C_4^{-2} + 12\sqrt{14}.C_6^{-2} +$ still higher order contributions$)/r_{max}^2$....(5)

Here J_{45} is the Jackson crossed-cylinder power. Use of only the first, C_2^{-2}, term in the bracket would give the least-squares value of J_{45}.

Clearly the other second-order polynomial

$Z_2^{2} = \sqrt{6}\rho^2\cos2\theta$

is also a crossed cylinder, but its axes are at 90/180, since $\cos2\theta = +1$ when $\theta = 0, 180$, and $= -1$ when $\theta = 90, 270$. The corresponding paraxial crossed-cylinder power, including the contributions of higher as well as second-order polynomials, is:

$J_{180} = -(2\sqrt{6}\ C_2^{2} - 6\sqrt{10}\ C_4^{2} + 12\sqrt{14}\ C_6^{2}$ + still higher order contributions$)/r_{max}^2$ .....(6)

As was shown by several authors,[13,14,17] the conventional sphero-cylindrical correction in the form S/C x α, with a negative cylinder, is then given by:

$C = -2\sqrt{(J_{180}^2 + J_{45}^2)}$ ....(7)
$S = M - C/2$ ....(8)
$\alpha = [\tan^{-1}(J_{45}/J_{180})]/2$ ....(9)

If J_{180} is zero, the equation for α gives an indeterminate result. In this case, if $J_{45}<0$, the $\alpha = 135$, and if $J_{45} \geq 0$, $\alpha = 45$.

The only other thing we need to worry about is keeping the axis of the cylinder in the clinical range 0-180 degrees. This is achieved by applying the following rules:

If $J_{180} < 0, \alpha = \alpha + 90$
If $J_{180} \geq 0\ \alpha = \alpha + 180$

We can see that these formulae allow us to 'subtract out' the effects of second-order sphero-cylindrical defocus from the wavefront map to leave just the higher-order (third-order and above) Zernike aberrations. Usually the aberrometer software will give a sphero-cylindrical ('least-squares') correction based only on the second-order coefficients. Equations (7), (8) and (9) are used, with the values of M, J_{45} and J_{180} being derived from equations (4), (5) and (6), with inclusion of only the second-order coefficients (ie C_2^0, C_2^{-2} and C_2^2 as appropriate).

One problem with the derived correction is that almost all objective aberrometers make their measurements in the infra-red. To compensate for the eye's longitudinal chromatic aberration and give a correction appropriate for a visible monochromatic wavelength (usually in the yellow or green) it is common for manufacturers to add an appropriate negative sphere (around -0.40D) to the estimated infra-red correction: there may also be an extra focus allowance to account for the fact that the light used in the aberrometry measurements may not reflect from the same plane as that of the retinal receptors. It may be that a manufacturer includes a vertex distance correction to give the prescription in the spectacle or some other plane. Other factors that may be allowed for include the fact that clinical refractions are carried out at 6m (vergence -0.17D) and that in the subjective method a 'least-minus' criterion which deliberately leaves the eye slightly myopic and relies on ocular depth-of-focus to give satisfactory vision at infinity is often used. In total, then, the output refraction may include a 'fudge factor' to bring it as closely into line

TABLE 1

Zernike polynomials up to the fourth order (ANSI, 2004) in terms of the polar coordinates (ρ, θ): j is the single-figure index, n the radial order and m the azimuthal frequency. Lists including still higher orders can be found elsewhere[7,12]

j	n	m	Z_n^m	Description
0	0	0	1	Piston
1	1	-1	$2\rho\sin\theta$	Vertical tilt
2	1	1	$2\rho\cos\theta$	Horizontal tilt
3	2	-2	$\sqrt{6}.\rho^2\sin2\theta$	Astigmatism (45/135)
4	2	0	$\sqrt{3}.(2\rho^2 - 1)$	Spherical defocus
5	2	2	$\sqrt{6}.\rho^2\cos2\theta$	Astigmatism (90/180)
6	3	-3	$\sqrt{8}.\rho^3\sin3\theta$	Oblique trefoil
7	3	-1	$\sqrt{8}.(3\rho^3 - 2\rho)\sin\theta$	Vertical coma
8	3	1	$\sqrt{8}.(3\rho^3 - 2\rho)\cos\theta$	Horizontal coma
9	3	3	$\sqrt{8}.\rho^3\cos3\theta$	Horizontal trefoil
10	4	-4	$\sqrt{10}.\rho^4\sin4\theta$	Oblique quatrefoil
11	4	-2	$\sqrt{10}.(4\rho^4 - 3\rho^2)\sin2\theta$	Secondary astig. (oblique)
12	4	0	$\sqrt{5}.(6\rho^4 - 6\rho^2 + 1)$	Spherical aberration
13	4	2	$\sqrt{10}.(4\rho^4 - 3\rho^2)\cos2\theta$	Secondary astig (vert/horiz)
14	4	4	$\sqrt{10}.\rho^4\cos4\theta$	Quatrefoil