Spherical aberration has always been considered to be important in the case of the eye, since it is the only monochromatic aberration that would be expected to occur on the axis of a centred optical system. In practice in individual real eyes, which almost always lack a true optical axis, coma and the other aberrations are often equally or more important. Interestingly, however, whereas the values of the individual higher-order Zernike coefficients in a population of normal eyes tend to be randomly distributed about zero, so that their mean is zero, the fourth-order spherical aberration coefficient C_4^0 is systematically biased in the positive direction. The population mean for C_4^0, therefore, is positive, corresponding to under-corrected spherical aberration, a typical finding[23] being a value of about 0.15 microns for a 5.7mm pupil diameter. How can we translate such findings into their dioptric equivalents?

From Table 1, for the relevant fourth-order Zernike polynomial, the wavefront aberration is:

$$W_r = C_4^0.\sqrt{5}.(6(r/r_{max})^4 - 6(r/r_{max})^2 + 1)$$

Note the absence of θ in this expression, implying rotational symmetry about the pupil centre. The second-order and constant terms of the polynomial represent balancing defocus and piston terms and can be ignore as far as the changes in power with r, which represent the spherical aberration, are concerned.

Differentiating the r^4 term gives:

$$dW_r/dr = (24.\sqrt{5}.C_4^0.r^3)/r_{max}^4$$

But each zone of the wavefront has an effective focal length f and power F. Since the rays are everywhere perpendicular to the wavefront, ignoring signs, the slope of the wavefront in the zone is the reciprocal of the slope of the corresponding rays. Hence

$$dWr/dr = (24.\sqrt{5}.C_4^0.r^3)/r_{max}^4 = r/f = rF$$

where f and F are the focal length and power of the zone respectively.

$$\text{ie } F = (24.\sqrt{5}.C_4^0.r2)/r_{max}^4$$

$$\text{and } F/r^2 = (24.\sqrt{5}.C_4^0)/r_{max}^4 \text{ dioptres/mm}^2$$

where the coefficient is in microns.

It is again usual to think in terms of corrections, so that, in dioptric terms, the primary spherical aberration equivalent to C_4^0 is:

$$F_{sa}/r^2 = -(24.\sqrt{5}.C_4^0)/r_{max}^4 \text{ dioptres/mm}^2 \quad ....(11)$$

Thus the C_4^0 value of 0.15 microns for a 5.7mm diameter pupil as found by Porter *et al*[23] corresponds to:

$$F_{sa}/r^2 = -(24.\sqrt{5}.0.15)/(2.85)^4 \text{ dioptres/mm}^2 = -0.12\text{D/mm}^2$$

This in turn is equivalent to under-corrected spherical aberration of about 1.08D at the edge of a 6mm pupil, which agrees quite well with traditional values in the older literature.[24]

If required, it is possible to extend the basic arguments and equations to include the Z_6^0 and Z_8^0 polynomials, which obviously represent spherical aberration which varies with the sixth and eighth powers of r, as well as including fourth-order spherical aberration and other terms. Inclusion of these terms might well be important after, eg myopic refractive surgery by photorefractive keratectomy (PRK) or laser assisted keratomileusis (Lasik) where only the central zones of the pupil are flattened and the periphery may remain steep, so that power changes rapidly in the outer zones of a dilated pupil. For example, if we consider so-called secondary spherical aberration polynomial:

$$Z_6^0 = \sqrt{7}.(20\rho^6 - 30\rho^4 + 12\rho^2 - 1)$$

this contributes secondary spherical aberration (the ρ^6 term), primary spherical aberration (the ρ^4 term) and defocus (the ρ^2 term).

We have already seen how the defocus term modifies the overall spherical dioptric error (equation 4). The primary spherical aberration given by equation (11) is modified by the ρ^4 term to yield:

$$F_{sa}/r^2 = -(24.\sqrt{5}.C_4^0 - 120.\sqrt{7}.C_6^0)/r_{max}^4 \text{ dioptres/mm}^2 \quad ....(12)$$

The secondary spherical aberration contributed by the polynomial is:

$$F_{sa}/r^4 = -(120.\sqrt{7}.C_6^0)/r_{max}^6 \text{ dioptres/mm}^4 \quad ....(13)$$

DISCUSSION

We can see that it is possible to extract from the overall wavefront aberration of the eye a sphero-cylindrical correction based either only on the second-order Zernike coefficients (the least-squares correction) or on all the relevant coefficients (the paraxial correction). The work of Atchison *et al*[14] and Thibos *et al*[15] suggests that, if the Zernike coefficients are derived with relatively large pupils (around 6mm or above), the paraxial correction is more useful, but that if the Zernike coefficients are derived for a small pupil (eg 3mm) the least-squares, second-order correction may be adequate.

The 'equivalent defocus' is a useful approximate way of envisaging in dioptric terms the degree of blur associated with single Zernike coefficients or their combinations for a particular pupil size, although it must not be used too uncritically. It may sometimes be helpful to express the Zernike spherical aberration coefficients in dioptric terms, since these are used to specify ocular spherical aberration in most earlier work.

Finally it is worth reminding ourselves that a Zernike analysis, and the designs of most aberrometers, work best when the wavefront aberration varies fairly smoothly across the pupil. Aberrometers may have considerable problems when there are rapid changes in wavefront error between closely neighbouring areas of the pupil, as might happen after some refractive surgery procedures, as a result of the wear of bifocal or mutifocal contact lenses with sharply defined regions of differing power, or corneal abnormalities. In Hartmann-Shack designs, for example, aberrometers may not sample the pupil at sufficient density to identify small-scale features in the wavefront, or there may be excessive displacement of the image spots of the Hartmann-Shack array, resulting in spot overlap and confusion (see, for example[4,5,7]). High degrees of scatter or absorption in the eye media, for example as a result of cataracts, may make measurements unreliable or impossible. Even if the overall form of the wavefront is correctly deduced, description of highly-aberrated, irregular wavefronts requires the use of a very large number of Zernike coefficients and it is doubtful if these can be calculated accurately enough.

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