Calibration of Weighing Instruments
Measurement Uncertainty Expert Know-how

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1. Introduction

The metrology of measuring instruments is a critical component of any organization’s quality operations. The standout prerequisite for traceable and accurate weighing is the effective calibration of weighing instruments, covered under the company’s quality management system. It seems, however, that not all organizations have a thorough understanding of current metrological science. Unless specifically addressed by metrologists, it is still a widespread belief that calibrating a weighing instrument mainly consists of placing reference masses on the weighing platform with the objective of assessing the deviation between the indication and the mass value of the reference. Not everybody is aware that calibration is a process that establishes a relation between quantity values provided by measurement standards and corresponding indications, which is only complete if the contributing measurement uncertainties are taken into account.

With regards to non-automatic weighing instruments, a manifold of calibration guidelines exist on a national level, which are all based on the concepts described in the Guide of the Expression of Uncertainty in Measurement (GUM). However, on a global level, only one document remains: The EURAMET calibration guideline cg-18 "Calibration of non-automatic weighing instruments". This document will also act as basis for the future development of a US calibration guideline.

This article focuses on the test procedures and measurement uncertainty estimation of the EURAMET cg-18 calibration guideline, specifically highlighting its practical implications for applications in the laboratory and the production area. One appendix of the guideline introduces the so-called “minimum weight”, i.e. the smallest sample quantity required for a weighment to just achieve a specified relative accuracy of weighing. The minimum weight defines the lower boundary of the safe weighing range, and weighing quantities within the safe weighing range guarantees compliance with the required weighing process tolerance. In simple words, calibration and the subsequent interpretation of its data establishes minimum weight and safe weighing range, and ensures the user meets applicable quality requirements.
2. Basic concepts of calibration

Calibration is one of the key activities that must be performed periodically when instruments are used for quality relevant measurements. Internationally, there are many standards which stipulate this requirement, e.g. ISO9001, GMP regulations or standards concerned with food safety. While almost everybody working in quality control and quality assurance in the laboratory or in the production environment is familiar with the applicable requirements stipulated in these documents, there is no common understanding on the definition, the implementation and the specific activities that comprise calibration. Let us therefore start by establishing a common platform on what calibration is.

Calibration is a set of activities carried out on a measurement instrument to understand its behavior by establishing a relationship between known values (measurement standards) and the associated measured values (indications). The relationship consists of a deviation and its associated uncertainty. The “International Vocabulary of Metrology” (VIM) provides the official definition of calibration:

"Operation that, under specified conditions, in a first step, establishes a relation between the quantity values with measurement uncertainties provided by measurement standards and corresponding indications with associated measurement uncertainties and, in a second step, uses this information to establish a relation for obtaining a measurement result from an indication."

It is evident that the relation between the known and the measured values can only be established if the associated measurement uncertainties are derived. Unfortunately, in practice there is a widespread misconception in regards to calibration as most users do not consider measurement uncertainty when "calibrating" an instrument. Measurement uncertainty is defined in the "Guide to the Expression of Uncertainty in Measurement" (GUM):

"Parameter, associated with the result of a measurement that characterizes the dispersion of the values that could reasonably be attributed to the measurand."

Basically, measurement uncertainty describes how far away from the true value a measurement result reasonably might be. The measurement results and the estimated uncertainty are usually documented in so-called calibration certificates. ISO/IEC 17025 specifies the general requirements for the competence of laboratories to carry out tests and/or calibrations. An accreditation by an accreditation body is the formal process that certifies that a calibration laboratory is competent and fulfills the requirements stipulated by ISO/IEC 17025.

Before detailing how weighing instruments can be calibrated and their respective measurement uncertainty estimated, we would like to emphasize another misconception in regards to calibration: Besides calibration, an instrument can also be adjusted. Adjustment is defined in the “International Vocabulary of Metrology” (VIM) as follows:

"Set of operations carried out on a measuring system so that it provides prescribed indications corresponding to given values of a quantity to be measured."

In other words, when adjusting an instrument, its indications are modified in a way so that they correspond – as far as possible – to the quantity values of the measurement standards applied. Unfortunately, many users apply the words calibration and adjustment interchangeably, incorrectly or even randomly. Quite often, they talk about calibrating a weighing instrument, however they mean adjusting it. The VIM also emphasizes this by stating:

"Adjustment of a measuring system should not be confused with calibration, which is a prerequisite for adjustment. After an adjustment of a measuring system, the measuring system must usually be recalibrated."

This statement highlights another important aspect of calibration: Before an instrument is adjusted, it must be first calibrated in order to understand – and document – its behavior. This is specifically important in order not to break the traceability chain of preceding measurements on the device. Equally after an adjustment, the instru-
ment must usually be recalibrated. Quite often, users talk about an "as found" calibration, i.e. a calibration of the instrument before any modification (adjustment) is carried out, and about an "as left" calibration, i.e. a calibration of the instrument after any necessary adjustment and/or repair has been carried out.

Besides calibration, measuring instruments can also be verified. Usually, instruments need to fulfil predefined requirements, quite frequently expressed as tolerances. The "International Vocabulary of Metrology" (VIM) defines verification as follows:

"Provision of objective evidence that a given item fulfils specified requirements."

While calibration only establishes the relationship between measurement standards and indications ("how well the instrument performs"), verification assesses the instrument on whether or not it meets specific requirements ("does the instrument perform well enough"). Usually, the outcome of verification is a "pass" or a "fail", while calibration itself does not provide an assessment. However, the calibration procedure is in many cases the starting point for a subsequent assessment of the results. Therefore, it is common practice to document the assessment as an annex to the calibration certificate. Tolerances can come from a variety of sources. With respect to weighing instruments, aside from the manufacturer who specifies tolerances for each balance or scale model, international or national testing recommendations and handbooks for weighing instruments used for applications involving commercial transactions (like OIML R76-1 or HB44) as well as industry specific regulations (like USP General Chapter 41) also specify tolerances. However, even more importantly, the user also needs to specify weighing tolerances that assure that the instrument performs well enough to fulfil his specific process requirements. In view of the application of the weighing instruments, these tolerances are the most important ones as they have a direct impact on the quality of the final product.

### 3. Calibration of non-automatic weighing instruments (NAWI)

Non-automatic weighing instruments have become ubiquitous for applications in the laboratory, on the production floor and in many other business areas (retail, pre-packaging, vehicle weighing, etc.). Due to their importance, calibration guidelines have been developed by many different organizations, especially since electronic weighing instruments were established on the market. Traditionally, these calibration guidelines were applied on a national level as the underlying issuing organizations were either national metrology institutes, national accreditation bodies or other nationally recognized organizations.

As a consequence of an effort to harmonize the requirements for the calibration of NAWIs on an international level, the guideline EURAMET cg-18 "Guidelines on the calibration of non-automatic weighing instruments" was developed by the leading European metrology institutes. It is now widely applied by calibration laboratories in Europe, and many national accreditation bodies take it as the state-of-the-art reference for accreditation of calibration laboratories. The guide has been adopted by SIM (Sistema Interamericano de Metrología) and thus is formally recognized by the regional American metrology organizations. Furthermore, it is also applied by several calibration laboratories in Africa and Asia, however not yet systematically.

Due to its widespread use, the EURAMET cg-18 calibration guide is the most frequently used reference document for the calibration of NAWIs. There are recent activities in the US, triggered by ASTM, who are interested in taking over the methodology of cg-18 and transposing it into an ASTM standard, which could potentially serve as a future national calibration guide for the US.
4. Calibration Procedure of EURAMET cg-18

4.1 Measurement Methods and Measurement Results

Usually, a repeatability test, a test for errors of indication and an eccentricity test are performed to assess the performance of the weighing instrument. With respect to assessing the normal use of the instrument or evaluating the performance under special conditions of use, EURAMET cg-18 allows for a flexible execution of the tests; however adherence to specific minimum requirements of the tests is stipulated as explained in the following paragraphs.

4.1.1 Repeatability test

Usually, a test load of about $0.5 \cdot \text{Max}$ to $\text{Max}$ is quite common ($\text{Max}$ stands for the maximum capacity of the instrument). However, this is often reduced for instruments where the test load would amount to several 1000 kg. For multiple range and multi-interval instruments, a load below and close to the capacity of the range/interval with the smallest scale interval $d$ may be sufficient. A special value for the test load may be agreed where this is justified in view of a specific application of the instrument. An example would be weighing standards or samples on analytical and micro balances where the typical quantity that is weighed is at the low end of the measurement range. Here, a small repeatability test load at the lower end of the weighing range may be agreed. The test load should, as far as possible, consist of one single body. A strict requirement is that the load has to be applied at least 5 times, and at least 3 times where the load is or exceeds 100 kg.

Repeatability is quantified by calculating the standard deviation of the repeated measurements:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (I_i - \bar{I})^2}$$

with $n$ being the number of repeated weighings, $I_i$ representing the individual indications and $\bar{I}$ being the mean value of the indications.

4.1.2 Test for errors of indication

This test requires at least five test points, distributed fairly evenly over the weighing range of the instrument. Note that zero is considered to be a test point. The reason is that a measurement uncertainty can also be allocated to zero. Consequently, an error of indication test with four physical test loads, plus the zero, fulfills the minimum requirements of EURAMET cg-18. The individual errors of indications $E_j$ are calculated as:

$$E_j = I_j - m_{\text{ref},j}$$

Usually, the reference value of mass $m_{\text{ref},j}$ of the test loads is approximated to its nominal value $m_{N,j}$ or its conventional value $m_{c,j}$. $I_j$ represents the individual indications of the error of indication test points.

4.1.3 Eccentricity test

The test is carried out with a test load of about $\text{Max}/3$ or higher. Depending on the shape of the load receptor, the number of test points might vary. For rectangular and round platforms, usually four positions and the center are taken as test points. From the indications $I_i$ obtained in the different positions, the differences $\Delta I_{\text{ecc},i}$ are usually calculated as:

$$\Delta I_{\text{ecc},i} = I_i - I_j$$
with \( I_i \) representing the indications at the different positions of the load outside the center, and \( I_c \) normally being the center reading (if the load can be placed in the center of the platform). For estimation of the uncertainty the largest \( \Delta I_{ecc,i} \) (as absolute value) will be taken into account, which will be abbreviated \( |\Delta I_{ecc,i}|_{max} \).

4.2 Standard Uncertainty for Discrete Values

The first step with regards to estimating the measurement uncertainty is deriving the so-called standard uncertainty for discrete values. The basic formula for that step is:

\[
E = I - m_{ref}
\]

with the related variance:

\[
u^2(E) = u^2(I) + u^2(m_{ref})
\]

For better legibility, the index \( j \) was omitted. Note that above formulas apply for every error of indication test point, including zero. This means that for every error of indication test point the associated standard uncertainty is derived individually. This is graphically indicated in figure 1.

As can be seen from the above equation, the standard uncertainty for discrete values comprises of two main sources: \( u(I) \), the standard uncertainty of the indication and \( u(m_{ref}) \), the standard uncertainty of the reference mass.

This is also evident from the definition of calibration as per the VIM, which states that the quantity values provided by measurement standards as well as the corresponding indications are associated with measurement uncertainties.

4.2.1. Standard Uncertainty of the Indication

The standard uncertainty of the indication comprises of four individual contributions, which account for the rounding error of the no-load indication, the rounding error of the indication at load, the repeatability and the eccentricity of the weighing instrument. Note that the rounding error is taken into account twice as any indication \( I \) related to a
test load is the difference of the indications $I_L$ at load and $I_0$ at no-load. The standard uncertainty of the indication is obtained by the following formula:

$$u^2(I) = \frac{d_0^2}{12} + \frac{d_L^2}{12} + s^2(I) + u_{rel}^2(\delta I_{ecc})^2$$

### a. Rounding error of the no-load indication and the indication at load

The variances of the rounding errors have been derived based on the assumption of a rectangular distribution of the unrounded (analogue) measurement values\(^1\). In other words, it was assumed that the probability of occurrence of any unrounded measurement value (which is not indicated on the instrument) is constant within the limits of $\frac{d_0}{2}$ or $\frac{d_L}{2}$, respectively. The standard deviation of a rectangular distribution is:

$$\frac{d_0}{2\sqrt{3}} \approx 0.29d_0 \quad \text{or} \quad \frac{d_L}{2\sqrt{3}} \approx 0.29d_L,$$

leading to the variances indicated above in the formula. In case that the scale interval at no-load and at load is the same (abbreviated with $d$), the total uncertainty accounting for the rounding error of the indication would thus be derived as:

$$u^2_{\text{rounding}} = 2 \cdot \frac{d^2}{12} \quad \text{leading to} \quad u_{\text{rounding}} = \frac{d}{\sqrt{6}} \approx 0.41d.$$

This value is also known from other regulations (e.g. USP General Chapter 41) and constitutes the lowest standard uncertainty of the indication that can be realized on a weighing instrument for the case that the standard deviation of the repeatability is zero and the uncertainty related to eccentricity is neglected.

### b. Repeatability

Normally, a single repeatability test is carried out during calibration. When estimating measurement uncertainty, the standard deviation $s$ is considered as being representative for all indications of the instrument, and is taken as the uncertainty contribution of repeatability. In case that more than one repeatability test is carried out, EUR-AMET cg-18 describes specific rules how to apply the standard deviations within the weighing range(s) of the instrument.

### c. Eccentricity

The standard uncertainty accounting for eccentricity $u_{rel}(\delta I_{ecc})I$ is approximated as a linear function of the indication $I$. The proportionality factor, the relative uncertainty $u_{rel}(\delta I_{ecc})$ is given by:

$$u_{rel}(\delta I_{ecc}) = \frac{|\delta I_{ecc}|_{\text{max}}}{2U_{ecc}\sqrt{3}}$$

with $L_{ecc}$ being the test load applied.

#### 4.2.2. Standard Uncertainty of the Reference Mass

The standard uncertainty of the reference mass usually comprises of four individual contributions, which account for the tolerance or the uncertainty of the reference mass, air buoyancy, a possible drift of the reference mass since its last calibration and convection effects due to potential temperature differences between the reference mass and the instrument.

$$u^2(m_{\text{ref}}) = u^2(\delta m_c) + u^2(\delta m_B) + u^2(\delta m_D) + u^2(\delta m_{\text{conv}})$$

\(^1\) A rectangular distribution is assumed for most of the contributions to measurement uncertainty. As the rectangular distribution is explained for the case of rounding, it will not be detailed individually anymore in the following chapters.
a. Tolerance / uncertainty of the reference mass

Whether the tolerance or the uncertainty of the reference mass is taken as a contribution to the above formula depends on whether the nominal or the conventional value is taken as reference mass for the calculation of the individual errors of indication. If the nominal value \( m_N \) is used, then the weight tolerance \( Tol \) is taken into account, leading to:

\[
u(\delta m_N) = \frac{Tol}{\sqrt{3}}\]

Note that OIML R111-1\(^9\) and ASTM E617\(^{10}\) use the term "maximum permissible error" for the tolerance, for which quite frequently the abbreviation \( mpe \) is used.

If the conventional value \( m_c \) is applied, then the uncertainty \( U \) and the respective coverage factor \( k \) of the weight calibration certificate is taken into account, leading to:

\[
u(\delta m_c) = \frac{U}{k}\]

It is evident that the contribution of \( u(\delta m_c) \) can be minimized if the conventional value \( m_c \) instead of the nominal value \( m_N \) is applied. Where a test load consists of more than one standard weight, the standard uncertainties are added arithmetically not by sum of squares, to account for assumed correlations of the weights.

b. Buoyancy

In general, air buoyancy can be corrected if the density \( \rho \) of the reference mass and the air density \( \rho_a \) at the time of calibration of the weighing instrument are known. Normally, however, the air density is not measured during calibration of the weighing instrument, so that an air buoyancy correction is not carried out frequently in practice. In these cases, the (unknown) buoyancy correction is taken into account as an intrinsic part of the buoyancy uncertainty.

For this scenario, two cases have to be distinguished, one with the instrument being adjusted immediately before calibration, and the other with the instrument not being adjusted before calibration. If the instrument is adjusted immediately before calibration, the respective buoyancy uncertainty is minimized as a potential change in buoyancy due to different air densities at calibration and adjustment is removed. If the instrument is adjusted independent of the calibration, worst-case assumptions should be made in regards to the potential air density variation between adjustment and calibration. If conformity of the standard weights with OIML R111-1 is established, the following worst-case uncertainties can be derived:

If the instrument is adjusted immediately before calibration:

\[
u(\delta m_B) \approx \frac{mpe}{4\sqrt{3}}\]

If the instrument is not adjusted before calibration (thereby assuming an air density variation of 10% of the reference density of air \( \rho_0 \) between adjustment and calibration):

\[
u(\delta m_B) \approx (0.1 \frac{\rho_0}{\rho_c} m_N + \frac{mpe}{4})/\sqrt{3}\]

Especially the last formula constitutes a very conservative approach to estimating uncertainty due to buoyancy, and if there is evidence that air density variations are smaller than 0.1 \( \rho_0 \) this value should be substituted by a less conservative value.

\(^9\) Recourse is taken to section 10 of OIML R111-1. The density of the material for weights shall be such that a deviation of 10% from the specified air density (1.2 kg/m\(^3\)) does not produce an error exceeding one quarter of the maximum permissible error.
c. Drift of the reference mass

A possible drift of the reference mass \( m_c \), since its last calibration can be assumed based on the difference in \( m_c \) evident from consecutive calibration certificates of the standard weights. The drift may be also estimated in view of the quality of the weights, and frequency and care of their use, to a multiple (expressed by a factor \( k_D \)) of their expanded uncertainty \( U(\delta m_c) \). It is not advised to apply a correction to the reference mass, but include the potential drift in the uncertainty, which is given by:

\[
u(\delta m_c) = \frac{k_D U(\delta m_c)}{\sqrt{3}}\]

The factor \( k_D \) is usually chosen between 1 and 3.

For weights conforming to OIML R111-1 or ASTM E617, an upper limit \( mpe / \sqrt{3} \) for the uncertainty contribution due to the drift of the reference mass can be applied, provided subsequent weight calibrations confirm that the mpe of the applicable weight class is adhered to.

d. Convection effects

Where weights have been transported to the calibration site they may not have the same temperature as the instrument and its environment. A temperature difference leads to a change of the apparent mass \( \Delta m_{conv} \) of the weights due to viscous friction at their surface induced by air flow originating from convection. This effect should be taken into account by either allowing the weights to acclimatize to the extent that the remaining change \( \Delta m_{conv} \) is negligible in view of the uncertainty of the calibration, or by considering the possible change of indication in the uncertainty budget. The effect may have to be considered for weights of high accuracy class as OIML E2 or F1 weights. It is not advised to apply a correction, but include the potential convection effects in the uncertainty, which is given by:

\[
u(\delta m_{conv}) = \frac{\Delta m_{conv}}{\sqrt{3}}\]

Appendix F of EURAMET cg-18 provides further information and tables to derive \( \Delta m_{conv} \).

4.2.3. Summary – Standard Uncertainty for Discrete Values

Summarizing the contributions as described above, the standard uncertainty for discrete values \( u(E) \) can be derived as follows:

\[
u^2(E) = \frac{d_b^2}{12} + \frac{d_a^2}{12} + s^2(I) + u^2(\delta m_{acc})^2 + u^2(\delta m_c) + u^2(\delta m_g) + u^2(\delta m_{conv}) + u^2(\delta m_{drift})\]

Some of the contribution factors to the standard uncertainty might be relatively small, depending on the instrument and the accuracy class of the reference masses under consideration. It is within the responsibility and the competence of the calibration laboratory to neglect these contributions if appropriate.

4.3. Expanded Uncertainty at Calibration

After having derived the standard uncertainty at calibration, this uncertainty must be expanded by the coverage factor \( k \) which is chosen in a way such that the expanded uncertainty of measurement has a coverage probability of 95.45 %.

\[ U(E) = ku(E) \]

This means that the expanded uncertainty shall ensure that the true value – which is not known – lies with a probability of at least 95.45 % within the interval ‘measured value ± expanded measurement uncertainty’.
For the case that a normal (Gaussian) distribution can be attributed to the error of indication, and the standard uncertainty is of sufficient reliability, \( k = 2 \) applies. Normal distribution may be assumed where several uncertainty components, each derived from "well-behaved" distributions (normal, rectangular or similar) contribute to \( u(E) \) in comparable amounts. This is normally the case when calibrating NAWIs. Sufficient reliability can be assumed, e.g. if during a repeatability test a load is applied not less than 10 times.

For the case that a normal distribution can be attributed to the error of indication, but \( u(E) \) is not sufficiently reliable, then the coverage factor is determined mathematically as per concepts described in Appendix B of EURAMET cg-18 and the GUM. This case usually applies when during the repeatability test a load is applied less than 10 times. As a consequence of the smaller number of repeated weighings, the coverage factor will be larger than 2. The coverage factor depends on the effective degrees of freedom \( v_{\text{eff}} \), determined with the Welch-Satterthwaite formula:

\[
v_{\text{eff}} = \frac{u_1^4(E)}{\sum_{i=1}^{N} \frac{u_i^4(E)}{v_i}}
\]

where \( u_i(E) \) are the contributions to the standard uncertainty \( u(E) \) and \( v_i \) represents the effective degrees of freedom of the standard uncertainty contribution \( u_i(E) \). The coverage factor is subsequently calculated with the t-distribution or Student’s distribution.

4.4. Uncertainty of a Weighing Result

4.4.1. Interpretation of Calibration Data

After having performed the step of calculating the expanded uncertainty, the calibration itself is formally completed. However, the data derived so far is of reduced value for the user as three sources of interpretation are missing:

- Behavior of the instrument in-between the selected error of indication test points
- Estimation of the measurement uncertainty in normal usage
- Assessment of the instrument against specific requirements such as weighing process tolerances or specifications

The calibration data offers a restricted set of information as they derive the measurement uncertainty only for a very limited amount of error of indication test points. Selecting more error of indication test points does not overcome this obstacle as this still constitutes a very limited picture of the real behavior and increases the time (and cost) for calibration. Furthermore, calibration only assesses the behavior of the instrument at the time of calibration, but not during normal usage. Any potential influence on the instrument that occurs before or after the calibration cannot be taken into account during the calibration itself. However, when assessing the instrument against specific requirements such as weighing process tolerances or specifications, the normal usage should – as far as possible – be taken into account.

EURAMET cg-18 offers detailed information on how to interpret calibration data, i.e. on how to derive the so-called standard uncertainty of a weighing result \( u(W) \). The standard uncertainty of a weighing result takes into account the normal usage of the instrument and allows estimation of the measurement uncertainty for any quantity of material which is placed on the weighing instrument during normal use of the instrument. The uncertainty of a weighing result is frequently used for the assessment against weighing process requirements (tolerances) as e.g. in respect to minimum weight. This specific assessment is detailed later in this paper.

The uncertainty of a weighing result comprises of three different sources:

- The uncertainty assuming the reading was taken under the same conditions as those prevailing at calibration; the uncertainty is typically approximated with a linear equation over the whole measurement range, based on the error of indication test points and their uncertainties
• The uncertainty associated with environmental influences differing from the time of calibration $u(\delta R_{\text{inst}})$, e.g. change in the characteristic of the instrument caused by a change in ambient temperature or a change in the characteristic since the time of calibration due to drift, or wear and tear.

• The uncertainty associated with the operation of the instrument in a different way as during calibration $u(\delta R_{\text{proc}})$, e.g. accounting for a net weighing result after taring the instrument.

Note that the uncertainty of a weighing result is not covered by the accreditation as it is based on an interpretation of the calibration results and is not directly and exclusively derived from the measurement values of the instrument which were taken during calibration.

The contributions to $u(W)$ are usually constant over the measurement range or depend linearly on the reading of the instrument $R$, and thus may be grouped in two terms $\alpha_W^2$ and $\beta_W^2$:

$$u^2(W) = \alpha_W^2 + \beta_W^2 R^2$$

For multi-interval and multiple range instruments, the uncertainty of a weighing result would be expressed per interval/range.

### 4.4.2. Expanded Uncertainty of a Weighing Results

The expanded uncertainty is determined by:

$$U(W) = k u(W)$$

For $U(W)$ the coverage factor $k$ will in most cases be equal to 2 even where the standard deviation $s$ is obtained from only a few repeated weighings, and/or where $k>2$ (at calibration) was stated in the calibration certificate. This is due to the large number of terms contributing to $u(W)$.

### 4.4.3. Errors Included in Uncertainty – The "Global Uncertainty"

It is common practice, even of utmost importance, to derive a "global uncertainty" $U_{gl}(W)$ which includes the errors of indication such that no correction has to be applied to the reading of a weighing result. In practice, applying corrections for the day-to-day work is inefficient and essentially almost impossible. In this case, the weighing result can be expressed as follows:

$$W = R \pm U_{gl}(W)$$

A common approach to calculate the global uncertainty is to add arithmetically the expanded uncertainty of a weighing result $u(W)$ and the absolute of the error of indication $E(R)$, reflecting possible correlations between these two terms. $E(R)$ is quite often approximated by a linear equation: $E(R) = a_1 R$, with $a_1$ being the linear regression coefficient, so that:

$$U_{gl}(W) = k \sqrt{\alpha_W^2 + \beta_W^2 R^2} + |a_1| R$$

In order to facilitate the understanding and the interpretation the result, $U(W) = k \sqrt{\alpha_W^2 + \beta_W^2 R^2}$ which is a rather complicated formula, is approximated by a linear equation, so that:

$$U_{gl}(W) \approx U(W = 0) + \frac{U(W = \text{Max}) - U(W = 0)}{\text{Max}} R + |a_1| R$$

Introducing the parameters $\alpha_{gl}$ and $\beta_{gl}$, the global uncertainty can be expressed as:

$$U_{gl}(W) = \alpha_{gl} + \beta_{gl} \cdot R$$
5. The Concept of Minimum Weight and Safe Weighing Range

Due to its importance in practice, the concept of minimum weight is included in EURAMET cg-18. Minimum weight is defined as follows:

"The minimum weight is the smallest sample quantity required for a weighment to just achieve a specified relative accuracy of weighing." \(^{11,12}\)

Consequently, when weighing a quantity representing minimum weight, \(R_{\text{min}}\), the relative measurement uncertainty at minimum weight \(U_{\text{rel}}(R_{\text{min}})\) equals the required relative weighing accuracy\(^{11,12}\), \(R_{\text{eq}}\), so that:

\[
U_{\text{rel}}(R_{\text{min}}) = \frac{U(R_{\text{min}})}{R_{\text{min}}} = R_{\text{eq}}
\]

This leads to the following relation that describes minimum weight:

\[
R_{\text{min}} = \frac{U(R_{\text{min}})}{R_{\text{eq}}}
\]

It is general practice for users to define specific requirements for the performance of an instrument (User Requirement Specifications). Normally they define upper thresholds for measurement uncertainty values that are acceptable for a specific weighing application. Colloquially users refer to weighing process accuracy or weighing tolerance requirements\(^{11,12}\). Very frequently users also have to follow regulations that stipulate the adherence to a specific measurement uncertainty requirement. Normally these requirements are indicated as a relative value, e.g. adherence to a measurement uncertainty of 0.1%.

For weighing instruments, usually the global uncertainty is used to assess whether the instrument fulfils specific user requirements. The global uncertainty is typically approximated by a linear equation as described in the previous chapter. The relative global uncertainty thus is a hyperbolic function and is defined as:

\[
U_{\text{gl,rel}}(W) = \frac{U_{\text{gl}}(W)}{R} = \frac{\alpha_{\text{gl}}}{R} + \beta_{\text{gl}}
\]

For a given tolerance requirement, \(R_{\text{eq}}\), only weighings with \(U_{\text{gl,rel}}(W) \leq R_{\text{eq}}\) fulfil the respective user requirement. Consequently only weighings with a reading of:

\[
R \geq \frac{\alpha_{\text{gl}}}{R_{\text{eq}} - \beta_{\text{gl}}}
\]

have a relative measurement uncertainty smaller than the specific requirement set by the user and are thus acceptable. The limit value, i.e. the smallest weighing result that fulfils the user requirement is:

\[
R_{\text{min}} = \frac{\alpha_{\text{gl}}}{R_{\text{eq}} - \beta_{\text{gl}}}
\]

and is called "minimum weight". Based on this value the user is able to define appropriate standard operating procedures that assure that the weighings he performs on the instrument comply with the minimum weight requirement, i.e. he only weighs quantities with higher mass than the minimum weight.

\(^{11}\) A relative accuracy is usually indicated in %

\(^{12}\) "Weighing accuracy" and "weighing tolerance" can be used interchangeably. In this paper, from now on, the wording "weighing tolerance" is used.
As measurement uncertainty of a weighing result and thus also the global uncertainty may be difficult to estimate due to specific environmental factors such as high levels of vibration, draughts, influences induced by the operator, etc., or due to specific influences of the weighing application such as electrostatically charged samples, magnetic stirrers, etc., a safety factor $SF$ is usually applied. The safety factor is a number larger than one, by which the user requirement $Req$ is divided. The objective is to ensure that the relative global measurement uncertainty is smaller than or equal to the user requirement $Req$, divided by the safety factor. This ensures that environmental effects or effects due to the specific weighing application that have an important effect on the measurement and thus might temporarily increase the measurement uncertainty above a level estimated by the global uncertainty, still allow – with a high degree of insurance – that the user requirement $Req$ is fulfilled.

$$U_{rel,\text{rel}}(W) \leq \frac{Req}{SF}$$

Consequently, the minimum weight based on the safety factor can be calculated as:

$$R_{\text{min},SF} = \frac{\alpha_{gt} \cdot SF}{Req - \beta_{gt} \cdot SF}$$

This leads to the definition of the safe weighing range: It is the range of the instrument, where the user can weigh safely, i.e. he fulfills the weighing tolerance requirement and adheres to the defined safety factor, see figure 2.

**Figure 2:** Safe weighing range for an industrial floor scale, derived from the calibration data and presented in an annex to a calibration certificate. The accuracy limit of the instrument, the so-called minimum weight, is the intersection point between relative measurement uncertainty and the required relative weighing tolerance. Weighing quantities in the red region results in non-compliance with the tolerance requirement, while weighing quantities in the green region ensures the tolerance requirement is fulfilled (safe weighing range). Weighing quantities in the yellow area fulfills the user requirements; however the safety factor is not adhered to.

The user is responsible for defining the safety factor depending on the degree to which environmental effects and the specific weighing application could influence the measurement uncertainty. Note that $R_{\text{min}}$ can vary between two subsequent calibrations due to potential performance changes of the instrument and varying environmental conditions. By means of application of an appropriate safety factor it is ensured that the smallest net weight, i.e. the smallest net quantity that the user intends to weigh on the instrument, is always larger than the minimum weight whenever the instrument is used. The behavior of minimum weight over time and the concept of the safety factor are illustrated in figure 3.
Note that the minimum weight and the safe weighing range refer to the net (sample) weight which is weighed on the instrument, i.e. the tare vessel mass must not be considered to fulfil the user requirement \( \text{Req} \). Therefore minimum weight is frequently called “minimum sample weight”.

We want to emphasize that for weighing small loads on analytical and microbalances the dominant contribution factor to the uncertainty stems from repeatability (expressed as the standard deviation \( s \) of a series of weighings). As for these laboratory applications the minimum weight typically is a very small sample size as compared to the capacity of the balance, the absolute uncertainty at the minimum weight can be approximated by \( \alpha_{gl} \). Furthermore, \( \alpha_{gl} \) only consists of the uncertainty contributions due to the rounding error of the indication and repeatability, with repeatability being the dominant factor for these types of instruments. Consequently, the minimum weight for applications when small loads are weighed on analytical and microbalances can be approximated:

\[
R_{\text{min}} = \frac{\alpha_{gl}}{\text{Req} - \beta_{gl}} \approx \frac{\alpha_{gl}}{\text{Req}} \approx k \cdot \frac{s}{\text{Req}}
\]

The coverage factor \( k \) is usually chosen as 2 as described previously.

An example for this simplified approach to minimum weight is presented in the revised USP General Chapter 41. In this regulation, the repeatability requirement is defined as:

"Repeatability is satisfactory if two times the standard deviation of the weighed value, divided by the desired smallest net weight, does not exceed 0.10 %." 

In other words, measurement uncertainty is reduced to repeatability which constitutes a valid approach for the affected weighing applications as typically analytical and microbalances are used to weigh small quantities of samples or standards. In the revised USP General Chapter 1251, the concept of minimum weight as described above is included for the first time in an official pharmaceutical compendium\(^{13} \). Taking the repeatability criterion of General Chapter 41, the respective minimum weight is derived as:

\[
R_{\text{min}} = \frac{k \cdot s}{\text{Req}} = \frac{2 \cdot s}{0.10\%} = 2000 \cdot s
\]
Conclusion

Calibration of measuring instruments is amongst the most important activities within any quality management system. Unfortunately, industry practices with respect to weighing instruments do not always appropriately reflect state-of-the-art concepts. The most evident shortcoming is the lack of a scientifically correct estimation of the measurement uncertainty, which is needed to assess whether the instrument under consideration fulfils predefined process tolerances. The EURAMET cg-18 calibration guideline is the most widespread reference document that details the methodology of deriving the measurement uncertainty of non-automatic weighing instruments. It not only includes information on the uncertainty at calibration, but also on the uncertainty of a weighing result which describes the performance of the instrument during day-to-day work and frequently serves as a basis for assessing the instrument against predefined tolerances.

A critical consequence of calibration is the concept of the minimum weight. By determining the minimum weight the user can assure compliance with his weighing requirements by weighing a sufficiently higher quantity of material than the minimum weight, expressed quantitatively by the safety factor. The minimum weight defines the lower boundary of the safe weighing range, and weighing quantities of material within the safe weighing range guarantees compliance with the required weighing process tolerance. In simple words, calibration and the subsequent interpretation of its data establishes minimum weight and safe weighing range, and ensures the user meets applicable quality requirements.
References


